

LOCAL DIP TRANSFORMATION FOR FAST SEISMIC HORIZON RECONSTRUCTION

Guillaume Zinck*, Marc Donias*, Jacques Daniel* and Olivier Laviolle*

*Université de Bordeaux, IPB, Laboratoire IMS CNRS UMR 5218, 351 cours de la Libération, 33405 Talence cedex, France.

{guillaume.zinck, marc.donias, jacques.daniel, olivier.laviolle}@ims-bordeaux.fr

ABSTRACT

We propose a fast method to reconstruct a seismic horizon with respect to a set of picked input points. The reconstruction domain is subdivided in quadrilateral domains which are determined from input points while the entire horizon is obtained part-by-part by juxtaposing independent partial reconstructions. Each quadrilateral domain is mapped onto a rectangular domain on which a non-linear partial derivative equation relied on local dip is solved by an iterative process based on a Poisson equation. The key point is the transformation of the local dip, which allows to carry out a direct Fourier method with a low computational cost.

Index Terms— Seismic horizon reconstruction, Poisson equation, Fast Fourier method, Local dip transformation

1. INTRODUCTION

To improve seismic data interpretation and understand geological processes, many recent numerical frameworks have been dedicated to seismic horizon reconstruction. A seismic horizon is a hypersurface of a seismic image which delimits geological layers. Applications scopes are various, like geological model building, reservoir characterization [1], chronostratigraphic interpretation [2] [3] or fault detection [4]. Some authors [5] [6] perform the reconstruction by an integration of the estimated local dip along three-dimensional (3-D) seismic data. Lomask *et al.* [7] consider a global approach through a two-dimensional (2-D) non-linear partial derivative equation (PDE) relied on local dip. The PDE is solved using a Gauss-Newton approach by an iterative algorithm whose crucial step is the resolution of a Poisson equation. In the case of complex geometries, Lomask and Guitton [8] proposed to take into account geological constraints such as picked points. Nevertheless, the proposed method carries out an iterative algorithm which implies that the computational cost is often prohibitive for large data volume.

In this paper, we present a fast approach based on Lomask's iterative algorithm [7] to reconstruct a horizon with respect to a set of picked input points. Considering the input points as corners of quadrangles, the reconstruction domain of the horizon is subdivided in quadrilateral areas. Parts of the horizon are then reconstructed independently from each other

on subdomains while the entire horizon is obtained by juxtaposing all reconstructed parts. Each quadrilateral domain is mapped onto a rectangular domain through a geometrical transformation. Instead of modifying the Poisson equation as described in standard methods [9] [10], the key point of our approach is the transformation of local dip: the Poisson equation is therefore solved by a direct Fourier method which guarantees a low computational cost. It can be noted that this approach is valid outside the seismic application scope to reconstruct an explicit surface in a finite-dimensional space such as fibrous composite images and can be extended to an implicit surface [11].

This article is organized as follows: section 2 introduces Lomask's horizon reconstruction algorithm, section 3 deals with a new fast reconstruction method on non-rectangular domains while the two last sections respectively describe the part-by-part horizon reconstruction and exhibit results.

2. HORIZON RECONSTRUCTION ALGORITHM

A seismic horizon can be considered as a curved segment in a 2-D space or as a surface in a 3-D space and is represented by a function f defined on a domain Ω . The function f is connected [7] to the tangent \mathbf{p} of the local dip¹ by a PDE:

$$\forall \mathbf{x} \in \Omega, \quad \nabla f(\mathbf{x}) = \mathbf{p}(\mathbf{x}, f(\mathbf{x})), \quad (1)$$

where ∇ denotes the gradient operator. In a 2-D (resp. 3-D) space, \mathbf{x} denotes a one-dimensional (1-D) variable x (resp. a two-dimensional variable (x_1, x_2)) while the local dip is a known one (resp. two)-dimensional vector giving the slope of the horizon tangent line (resp. plane) compared to the space axis \vec{x} (resp. \vec{x}_1 and \vec{x}_2). The functions f and \mathbf{p} are respectively considered of class C^2 and C^1 .

The horizon is obtained by solving a constrained optimization problem:

$$f = \arg \min_{g \in C^2} \int_{\Omega} \|\nabla g(\mathbf{x}) - \mathbf{p}(\mathbf{x}, g(\mathbf{x}))\|^2 dx, \quad (2)$$

assuming that either the horizon boundary or points belonging to the horizon are known. Equation (2) is non-linear, thus

¹The tangent \mathbf{p} is previously estimated over the entire seismic data by a gradient field principal component analysis [12].

an iterative algorithm is used to solve it [7]. The horizon is initialized with a function f_0 and the iterative step is made of three parts : residual computation, update term computation and updating.

- *Residual computation:*

$$\forall \mathbf{x} \in \Omega, \quad \mathbf{r}_k(\mathbf{x}) = \nabla f_k(\mathbf{x}) - \mathbf{p}(\mathbf{x}, f_k(\mathbf{x})). \quad (3)$$

- *Update term computation:*

The update term δf_k is obtained by solving a Poisson equation

$$\Delta(\delta f_k) = -\text{div}(\mathbf{r}_k), \quad (4)$$

where Δ and div denote respectively the Laplace operator and the divergence operator.

If the horizon boundary is known, the Poisson equation is associated with boundary values called Dirichlet conditions:

$$\begin{aligned} \forall \mathbf{x} \in \partial\Omega, \quad \delta f_0(\mathbf{x}) &= f(\mathbf{x}) - f_0(\mathbf{x}) \\ \text{and } \delta f_k(\mathbf{x}) &= 0 \quad \forall k > 0, \end{aligned} \quad (5)$$

where $\partial\Omega$ denotes the boundary of the domain.

If a unique point belonging to the horizon is known [7], the derivative of the update term along the exterior normal \vec{w} to the boundary are assumed to be equal to zero. The Poisson equation is then associated with boundary values called Neumann conditions:

$$\forall \mathbf{x} \in \partial\Omega, \quad \nabla \delta f_k(\mathbf{x}) \cdot \vec{w}(\mathbf{x}) = 0, \quad (6)$$

where \cdot denotes the dot product operator. The problems (4)-(5) and (4)-(6) are called boundary problems.

If several points belonging to the horizon are known [8], the Poisson equation is associated with ‘‘inner’’ conditions:

$$\begin{aligned} \forall \mathbf{x} \in \Omega_c, \quad \delta f_0(\mathbf{x}) &= f(\mathbf{x}) - f_0(\mathbf{x}) \\ \text{and } \delta f_k(\mathbf{x}) &= 0 \quad \forall k > 0, \end{aligned} \quad (7)$$

where Ω_c denotes the union set of all known points. The problem (4)-(7) is called inner problem.

- *Updating:*

$$\forall \mathbf{x} \in \Omega, \quad f_{k+1}(\mathbf{x}) = f_k(\mathbf{x}) + \delta f_k(\mathbf{x}). \quad (8)$$

Usual stopping criteria [7] consider the norm of the residual or a maximal number K of iterations.

The ability to compute the update term determines the computational efficiency of the reconstruction method. On a 1-D domain and a 2-D rectangular domain, fast Fourier algorithms [13] can be applied to solve the boundary problems. The update term is computed in one step:

$$\delta f_k = \text{FT}^{-1} \left[\frac{\text{FT}[-\text{div}(\mathbf{r}_k)]}{\text{FT}[\Delta]} \right], \quad (9)$$

where FT and FT^{-1} denote respectively the Fourier transform and the inverse Fourier transform. However, the Fourier algorithms can not be carried out to solve the inner problem on the aforementioned domains. Iterative methods like descent direction approaches and relaxation algorithms are therefore proposed in the literature [14]. On 2-D non-rectangular domains (excepted on a disk), both problems lead to complex matrix inversions. For some particular domains diffeomorphic to a rectangular domain and called pseudo-rectangular domains in this paper, an alternative method is to map the physical domain Ω onto a rectangular computational domain Ω' by introducing a diffeomorphic transformation [9] [10]. On the domain Ω' , a differential operator with variable coefficients takes place of the Laplace operator in (4). Although less complex than those described previously on Ω , matrix methods to solve (4) on Ω' [15] [16] are relatively slow whereas Fourier algorithms are irrelevant.

3. FAST RECONSTRUCTION ON PSEUDO-RECTANGULAR DOMAINS

3.1. Local dip transformation

In this section, we present a fast horizon reconstruction on a pseudo-rectangular domain, assuming that either the horizon boundary or a unique point belonging to the horizon is known. Instead of replacing the Laplace operator in (4), the right term $-\text{div}(\mathbf{r}_k)$ is modified by a local dip transformation. The boundary problems can then be solved by a Fourier algorithm.

We propose to apply on (1) the diffeomorphic transformation \mathcal{F} which maps the pseudo-rectangular domain Ω onto a rectangular domain Ω' . The transformation is defined by

$$\forall \mathbf{x} \in \Omega, \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathcal{F}(\mathbf{x}) = \begin{bmatrix} \mathcal{F}_1(\mathbf{x}) \\ \mathcal{F}_2(\mathbf{x}) \end{bmatrix} \in \Omega'. \quad (10)$$

The gradient field of the function f is consequently relied on a vector field by a PDE:

$$\forall \mathbf{y} \in \Omega', \quad \nabla f(\mathbf{y}) = \mathbf{p}'(\mathbf{y}, f(\mathbf{y})), \quad (11)$$

where \mathbf{y} denotes the 2-D variable (y_1, y_2) . The 2-D vector \mathbf{p}' is the tangent of the transformed local dip, which gives the slope of the horizon tangent plane compared to the axis \vec{y}_1 and \vec{y}_2 of Ω' . It is expressed by

$$\mathbf{p}' = [J_{\mathcal{F}}^T]^{-1} \mathbf{p}, \quad (12)$$

where $[J_{\mathcal{F}}^T]^{-1}$ is the inverse of the transpose of the transformation Jacobian matrix $J_{\mathcal{F}}$ defined by

$$J_{\mathcal{F}}(\mathbf{x}) = \begin{bmatrix} \frac{\partial y_1}{\partial x_1}(\mathbf{x}) & \frac{\partial y_1}{\partial x_2}(\mathbf{x}) \\ \frac{\partial y_2}{\partial x_1}(\mathbf{x}) & \frac{\partial y_2}{\partial x_2}(\mathbf{x}) \end{bmatrix}. \quad (13)$$

Proof. Derivatives on Ω and Ω' [9] are connected by the relation

$$\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = [J_{\mathcal{F}}^T] \begin{bmatrix} \frac{\partial f}{\partial y_1} \\ \frac{\partial f}{\partial y_2} \end{bmatrix}. \quad (14)$$

According to equations (1) and (14),

$$[J_{\mathcal{F}}^T] \begin{bmatrix} \frac{\partial f}{\partial y_1} \\ \frac{\partial f}{\partial y_2} \end{bmatrix} = \mathbf{p}. \quad (15)$$

Multiplying both sides of (15) by $[J_{\mathcal{F}}^T]^{-1}$ leads to (12). \square

3.2. Example of the quadrilateral domain

A quadrilateral domain is an example of pseudo-rectangular domain. The diffeomorphic transformation \mathcal{F} introduced to map a quadrilateral domain onto a rectangular one is a homography defined by a 3×3 matrix $H = [h_{ji}]$ (see Fig. 1). The transformation is given by:

$$\forall \mathbf{x} \in \Omega, \begin{bmatrix} \mathcal{F}_1(\mathbf{x}) \\ \mathcal{F}_2(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} h_{11}x_1 + h_{12}x_2 + h_{13} \\ h_{31}x_1 + h_{32}x_2 + h_{33} \\ h_{21}x_1 + h_{22}x_2 + h_{23} \\ h_{31}x_1 + h_{32}x_2 + h_{33} \end{bmatrix}. \quad (16)$$

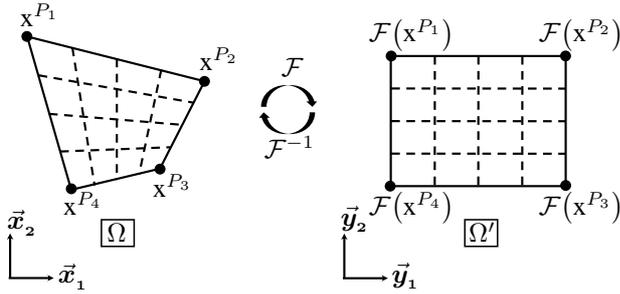


Fig. 1. Quadrilateral domain Ω and rectangular domain Ω' obtained by a homography \mathcal{F} .

4. PART-BY-PART RECONSTRUCTION

Given a set of input points and its convex envelope Γ , our part-by-part horizon reconstruction method consists of four steps:

1. Subdivision of Γ in quadrilateral domains Ω by considering the input points as corners of the domains.
2. Reconstruction of the horizon part on the corners and along the boundary $\partial\Omega$ of Ω .
3. Reconstruction of the horizon part on Ω .
4. Reconstruction of the entire horizon on Γ by juxtaposition of all reconstructed horizon parts.

A Delaunay triangulation is firstly performed for the input points [17]. Each triangle can secondly be subdivided in three quadrilateral elements by considering the center of mass as a shared corner [10]. As a result, the corners of the quadrangles are the input points, the midpoints of the sides and the centers of mass of the triangles while the boundaries are the sides and a part of the medians of the triangles (see Fig. 2). For each domain Ω , the four segments constituting the boundary $\partial\Omega$ are reconstructed in a 2-D space by the one-dimensional version of the horizon reconstruction algorithm even though the two-dimensional transformation defined by (12) is applied to the local dip tangent. These segments are then used as Dirichlet conditions in the algorithm to obtain the 2-D horizon part on Ω . As the method is clearly suboptimal, the choice of the computational domain size is a crucial step: Ω' must be large enough to avoid losing data accuracy but not too large to keep a low computational cost. The width and the height of Ω' are two independant sizes linked to the sizes of the two pairs of opposite sides of Ω . They can be taken as the shortest (Min) or the longest (Max) size as well as the arithmetic mean ($\bar{\sum}$) or the geometric mean ($\bar{\prod}$) of each pair. To reduce computational cost, each considered size can be replaced by the closest size which is optimal for a fast Fourier transform algorithm (for instance FFTW library [18]).

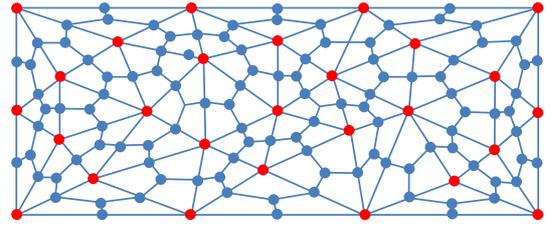


Fig. 2. Set of quadrangles with respect to 27 input points (red disks).

5. RESULTS

Part-by-part and Lomask's global optimization methods [8] are evaluated and compared on real seismic data ($1,000 \times 400 \times 350$, see Fig. 3). Complex geometries and convergent structures of the processed data result in an extremely noisy estimated dip, so a set of 14 input points are sequentially picked in critical regions (peaks, bassins, etc.) of the horizon to be reconstructed, starting from an initial set of 13 points. The number K of iterations is empirically fixed to 30 to reach convergence of both methods. For the part-by-part method, the 27 input points lead to 126 quadrangles (see Fig. 2). For the global optimization method, each update term computation through a direction descent approach requires 300 iterations and the algorithm has to be initialized with a function f_0 close to the solution. The function f_0 proposed in [8] is obtained from a horizon reconstructed over the entire domain by assuming that only one particular input point is known. The part-by-part and the global optimization

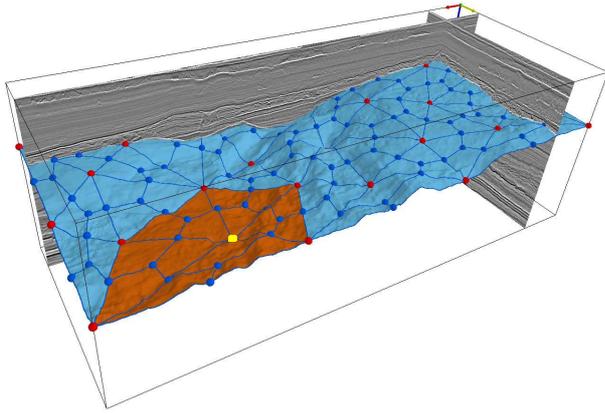
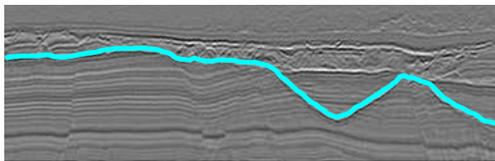
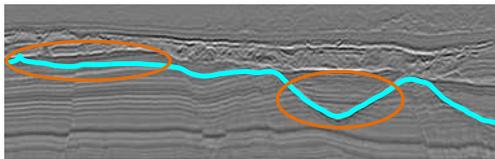


Fig. 3. Seismic data and part-by-part reconstructed horizon with respect to a set of 27 input points (red spheres). Only the orange quadrangles are recalculated when displacing the yellow point.



(a) Part-by-part reconstruction cross-section.



(b) Global optimization reconstruction cross-section.

Fig. 4. Part-by-part and global optimization reconstructions.

horizons are compared on a data cross-section for 27 input points (see Fig. 4). They are close to the visible horizon, which proves precision and noise robustness of both methods. Nevertheless, the global optimization horizon is not perfectly superimposed with the visible horizon on the left side, so the part-by-part method locally results in better quality horizons.

Computational costs for 27 input points are grouped in Table 1. In all cases, the cost of the part-by-part method is lower than the cost of the global optimization method, up to almost 30 times for the optimal shortest side size. This can principally be explained by two reasons. Firstly, the cost of the initialization step proposed in [8] is higher than the cost of the entire part-by-part horizon reconstruction without considering the local dip transformation step. Secondly, the update term is computed in one step in the part-by-part method whereas a large number of iterations is required in Lomask's one.

Adding or displacing vertically one input point causes a

Size of Ω'	Part-by-part method		Global optimization
	Normal size	Optimal size	
Min	3.3	2.7	79.1
Max	9.98	6.43	
\sum	5.82	4.26	
\prod	5.4	3.78	

Table 1. Computational times in seconds versus rectangular domains sizes for 27 inputs points.

Number of input points	Entire reconstruction	Incremental reconstruction
13	3.8	–
18	3.73	0.627
23	3.72	0.603
27	3.78	0.5

Table 2. Computational times in seconds of the part-by-part reconstruction versus number of input points.

reestimation of the entire horizon with the global optimization method, whereas only the quadrangles connected to the modified point need to be recalculated with the incremental part-by-part method (see Fig. 3). While the computational cost of the global optimization method does not depend on the number of input points, the costs of the entire and the incremental part-by-part reconstruction methods versus the number of points are presented in Table 2. As expected, the incremental part-by-part reconstruction is extremely fast compared to the entire reconstruction and its time decreases when the number of points increases. Incremental part-by-part method can consequently be considered as a real-time method which allows an interactive reconstruction of a seismic horizon.

6. CONCLUSION

We have developed a fast method to reconstruct a seismic horizon with respect to a set of input points. Our approach consists in a part-by-part reconstruction on quadrilateral subdomains. The key point is the transformation of the estimated local dip instead of the derivatives to solve a Poisson equation with a direct Fourier method, which guarantees a low computational cost. The horizons obtained for real seismic data prove accuracy and noise robustness of the method. They are close to the visible ones and to those reconstructed by a more time-consuming global optimization method. Moreover, our approach allows a real-time interactive reconstruction.

7. ACKNOWLEDGMENTS

The authors thank Total company for supporting this work, supplying seismic data and adding the implementation of their approach in the seismic interpretation platform SismageTM.

8. REFERENCES

- [1] J. Hoyes and T. Cheret, "A review of "global" interpretation methods for automated 3-D horizon picking," *The Leading Edge*, vol. 30, no. 1, pp. 38–47, 2011.
- [2] M. Donias, S. Guillon, P. Baylou, F. Pauget, and N. Keskes, "Method of chrono-stratigraphic interpretation of a seismic cross section or block," Elf Exploration Production, Patent US 0036294, November 2001.
- [3] V. Toujas, M. Donias, D. Jeantet, S. Guillon, and Y. Berthoumieu, "A robust framework for GeoTime cube," in *International Conference on Image Processing (ICIP)*. IEEE, 2008, pp. 1880–1883.
- [4] G. Zinck, M. Donias, S. Guillon, and O. Lavialle, "Discontinuous seismic horizon tracking based on a Poisson equation with incremental Dirichlet boundary conditions," in *International Conference on Image Processing (ICIP)*. IEEE, 2011, pp. 3385–3388.
- [5] N. Bienati and U. Spagnolini, "Traveltime picking in 3-D data volumes," in *Extended Abstracts, Session : 1-12*. 60th EAGE Meeting, 1998, pp. 98–112.
- [6] A. Blinov and M. Petrou, "Reconstruction of 3-D horizons from 3-D seismic datasets," *IEEE Transactions on Geoscience and Remote Sensing*, vol. 43, no. 6, pp. 1421–1431, 2005.
- [7] J. Lomask, A. Guitton, S. Fomel, J. Claerbout, and A. Valenciano, "Flattening without picking," *Geophysics*, vol. 71, no. 1, pp. 13–20, 2006.
- [8] J. Lomask and A. Guitton, "Flattening with geological constraints," in *Annual Meeting Expanded Abstracts*. Society of Exploration Geophysicists (SEG), 2006, pp. 1053–1056.
- [9] R.E. Bellman and J. Casti, "Differential quadrature and long-term integration," *Journal of Mathematical Analysis and Applications*, vol. 34, no. 2, pp. 235–238, 1971.
- [10] H. Zhong and Y. He, "Solution of Poisson and Laplace equations by quadrilateral quadrature element," *International Journal of Solids and Structures*, vol. 35, no. 21, pp. 2805–2819, 1998.
- [11] G. Zinck, M. Donias, and O. Lavialle, "N-dimensional surface reconstruction from a noisy normal vector field," in *European Signal Processing Conference (EUSIPCO)*. EURASIP, 2012, pp. 395–399.
- [12] K.. Marfurt, "Robust estimates of 3-D reflector dip and azimuth," *Geophysics*, vol. 71, no. 4, pp. 29–40, 2006.
- [13] R.W. Hockney, "A fast direct solution of Poisson's equation using Fourier analysis," *Journal of the ACM*, vol. 12, pp. 95–113, 1965.
- [14] A. D. Polyanin, *Handbook of Linear Partial Differential Equations for Engineers and Scientists*, Chapman & Hall/CRC, 2002.
- [15] H. Johansen and P. Colella, "A cartesian grid embedded boundary method for Poisson's equation on irregular domains," *Journal of Computational Physics*, vol. 147, no. 1, pp. 60–85, 1998.
- [16] R.J. Leveque and Z. Li, "The immersed interface method for elliptic equations with discontinuous coefficients and singular sources," *SIAM Journal on Numerical Analysis*, vol. 31, no. 4, pp. 1019–1044, 1994.
- [17] B. Delaunay, "Sur la sphère vide. A la mémoire de Georges Voronoi," *Bulletin de l'Académie des Sciences de l'URSS. Classe des sciences mathématiques et na*, vol. 6, pp. 793–800, 1934.
- [18] M. Frigo and S. G. Johnson, "FFTW: an adaptive software architecture for the FFT," in *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. 1998, pp. 1381–1384, IEEE.